Note: Throughout this test the symbol $i$ denotes $\sqrt{-1}$.

1. Test the following series for convergence or divergence. Decide for yourself which test is easiest to use.
   (a) (10 points)
   $$\sum_{n=1}^{\infty} \frac{n^5}{5^n}$$
   (b) (10 points)
   $$\sum_{n=2}^{\infty} \frac{1}{n^2 - n}$$

2. Use series to calculate
   (a) (10 points)
   $$\frac{d^4}{dx^4} \ln(1 + x^3) \text{ at } x = 0$$
   (b) (10 points)
   $$\frac{d^{10}}{dx^{10}} (x^8 \tan^2 x) \text{ at } x = 0$$

3. (10 points) Find the following limit using Maclaurin series.
   $$\lim_{x \to 0} \left( \frac{1}{x} - \frac{1}{e^x - 1} \right)$$
   Hint: First combine fractions. Then find the first term of the denominator series and the first term of the numerator series.

4. (20 points) Using $\exp i\theta = \cos \theta + i \sin \theta$, prove that
   $$\cos \theta + \cos 3\theta + \cos 5\theta + \ldots + \cos(2n - 1)\theta = \frac{\sin 2n\theta}{2 \sin \theta}$$
   Hint: For a complex number $z$,
   $$\sum_{k=0}^{n-1} z^k = \frac{1 - z^n}{1 - z}$$

5. (15 points) Find all values of $\sqrt[3]{-1}$.

6. (15 points) Find the disk of convergence of the series $\sum_{n=1}^{\infty} (z - 2i)^n / n$.

7. (10 points) Calculate the determinant
   $$\begin{vmatrix}
   \cos \theta & i \sin \theta \\
   i \sin \theta & \cos \theta
   \end{vmatrix}$$