1 (a) Consider the tight binding Hamiltonian in one dimension with a square well potential of depth $U$ applied to the zeroth site:

$$H = -U|0\rangle\langle 0| - \sum_{j=-\infty}^{\infty} \frac{V}{2} (|j+1\rangle\langle j| + |j\rangle\langle j+1|)$$

The square well potential can be thought of as the effect of an impurity on conduction band electrons. Calculate the difference in energy between the bound state of the impurity and the bottom of the conduction band.

(b) What is this in the limit of large $V/U$?

(c) Consider the continuum version of this problem, an electron in one dimension in the presence of a potential $-U\delta(x)$. Calculate the energy of the bound state.

(d) Calculate the effective mass $m^*$ of an electron at the bottom of the band in (a). Rewrite your answer for the energy difference in (b) in terms of $m^*$ and $U$. How does this compare with (c)?