Very small solid particles, called grains, exist in interstellar space. They are continually bombarded by hydrogen atoms of the surrounding interstellar gas. As a result of these collisions, the grains execute Brownian movement in both translation and rotation. Assume the grains are uniform spheres of diameter $7 \times 10^{-6} \text{cm}$ and density $1 \text{g/cm}^3$, and the temperature of the gas is $100 \text{K}$. Find the root mean square speed of the grains between collisions.

**Useful information:** $k_B = 1.38 \times 10^{-23} \text{J/molecule} \cdot \text{K}$. 
\[ d = 7 \times 10^{-6}, T = 100 \]

The equipartition theorem says \( \langle \text{TranslationalKineticEnergy} \rangle = (\text{number of degrees of freedom}) \times k_B T / 2 \)

The number of degrees of freedom are 3 per particle. So \( \langle \frac{1}{2}mv^2 \rangle = \frac{3}{2}k_B T \) which gives \( m = \rho V \) with \( V = \frac{4\pi}{3}r^3 \) with \( r = 7 \times 10^{-6}/2 = 3.5 \times 10^{-8} m \). Therefore \( m = 1.79594379825 \times 10^{-19} kg \).

So \( \langle v^2 \rangle = 3k_B T/m \).

So \( v_{rms} = \sqrt{\langle v^2 \rangle} = \sqrt{3 \times 1.38 \times 10^{-23} \times 100/(1.79594379825 \times 10^{-19})} = 0.152 m/s \).
Very small solid particles, called grains, exist in interstellar space. They are continually bombarded by hydrogen atoms of the surrounding interstellar gas. As a result of these collisions, the grains execute Brownian movement in both translation and rotation. Assume the grains are uniform spheres of diameter $7 \times 10^{-6} \text{cm}$ and density $1 \text{g/cm}^3$, and the temperature of the gas is $50 \text{K}$. Find the root mean square speed of the grains between collisions.

**Useful information:**  $k_B = 1.38 \times 10^{-23} \text{J/molecule} \cdot \text{K}$.
$d = 7 \times 10^{-6}, T = 50$

The equipartition theorem says \( \langle \text{TranslationalKineticEnergy} \rangle = (\text{number of degrees of freedom}) \times k_B T / 2 \)

The number of degrees of freedom are 3 per particle. So \( \langle \frac{1}{2} m v^2 \rangle = \frac{3}{2} k_B T \) which gives $m = \rho V$ with $V = (4\pi/3)r^3$
with $r = 7 \times 10^{-6} / 2 = 3.5 \times 10^{-8} m$. Therefore $m = 1.79594379825 \times 10^{-19} kg$.

So $\langle v^2 \rangle = 3k_B T / m$.

So $v_{rms} = \sqrt{\langle v^2 \rangle} = \sqrt{3 \times 1.38 \times 10^{-23} \times 50 / (1.79594379825 \times 10^{-19})} = 0.107 m/s$. 
Very small solid particles, called grains, exist in interstellar space. They are continually bombarded by hydrogen atoms of the surrounding interstellar gas. As a result of these collisions, the grains execute Brownian movement in both translation and rotation. Assume the grains are uniform spheres of diameter $5 \times 10^{-6} \text{ cm}$ and density $1 \text{ g/cm}^3$, and the temperature of the gas is $100^\circ \text{K}$. Find the root mean square speed of the grains between collisions.

**Useful information:** $k_B = 1.38 \times 10^{-23} \text{ J/molecule} \cdot \text{K}$. 

Please read the question carefully before attempting it. You will not be given any credit if you only write down the final answers. You must show your work.
\[ d = 5 \times 10^{-6}, T = 100 \]

The equipartition theorem says \( \langle \text{TranslationalKineticEnergy} \rangle = \text{(number of degrees of freedom)} \times k_B T / 2 \)

The number of degrees of freedom are 3 per particle. So \( \langle \frac{1}{2}mv^2 \rangle = \frac{3}{2}k_B T \) which gives \( m = \rho V \) with \( V = (4\pi/3)r^3 \) with \( r = 5 \times 10^{-6}/2 = 2.5 \times 10^{-8}m \). Therefore \( m = 6.5449846875 \times 10^{-20}kg \).

So \( \langle v^2 \rangle = 3k_B T/m \).

So \( v_{rms} = \sqrt{\langle v^2 \rangle} = \sqrt{3 \times 1.38 \times 10^{-23} \times 100/(6.5449846875 \times 10^{-20})} = 0.252m/s \).
Please read the question carefully before attempting it. You will not be given any credit if you only write down the final answers. You must show your work.

Very small solid particles, called grains, exist in interstellar space. They are continually bombarded by hydrogen atoms of the surrounding interstellar gas. As a result of these collisions, the grains execute Brownian movement in both translation and rotation. Assume the grains are uniform spheres of diameter $2 \times 10^{-6} \text{cm}$ and density $1 \text{g/cm}^3$, and the temperature of the gas is $150 \text{K}$. Find the root mean square speed of the grains between collisions.

Useful information: $k_B = 1.38 \times 10^{-23} \text{J/molecule} \cdot \text{K}$.
\[ d = 2 \times 10^{-6}, T = 150 \]

The equipartition theorem says \( \langle \text{Translational Kinetic Energy} \rangle = (\text{number of degrees of freedom}) \times k_B T / 2 \)

The number of degrees of freedom are 3 per particle. So \( \langle \frac{1}{2} m v^2 \rangle = \frac{3}{2} k_B T \) which gives \( m = \rho V \) with \( V = (4\pi/3)r^3 \) with \( r = 2 \times 10^{-6} / 2 = 1 \times 10^{-8} \) m. Therefore \( m = 4.1887902 \times 10^{-21} \) kg.

So \( \langle v^2 \rangle = 3k_B T / m. \)

So \( v_{rms} = \sqrt{\langle v^2 \rangle} = \sqrt{3 \times 1.38 \times 10^{-23} \times 150 / (4.1887902 \times 10^{-21})} = 1.22 m/s. \)